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CALCULATION OF THE TEMPERATURE FIELD IN LASER IRRADIATION OF A LAMINAR COMPOSITE

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Using the solution of a boundary-value problem of nonstationary heat conduction, we suggest an approach to determining the temperature in laser irradiation of the surface of a two-period laminar semiinfinite body.

Introduction. Heating of materials by local heat sources has come into widespread use in practice. Most common are processes in which the model of the object of heating is in the form of a semiinfinite body. Nonstationary temperature fields that appear as a result of the thermal action of a laser beam on the surface of a homogeneous half-space were studied in numerous works, reviews of which are presented in [1-3]. We investigate the influence of the relationship between the thermophysical parameters (the coefficients of thermal conductivity and thermal diffusivity) of the materials of a two-period semiinfinite laminar composite on the transient temperature processes occurring in it due to the heating of a free surface by a laser heat flux with a uniform or normal (Gaussian) distribution of intensity.

1. Statement of the Problem. We will consider a semiinfinite body consisting of a periodic system of two connected heterogeneous layers of thickness l_1 and l_2. The initial temperature of the composite is assumed to be zero. At the instant of time $t = 0$ we begin to heat the body surface by a laser heat flux distributed uniformly or normally in a circle of radius a. Outside this region, the surface of the half-space is thermally insulated, and the layers are in ideal thermal contact with each other.

In this statement the temperature field that appears in the composite is axisymmetric. Therefore we refer the body to a cylindrical coordinate system (r, z) with the origin at the center of the heating circle and the z axis directed into the composite.

In application to boundary-value problems of heat conduction and thermoelasticity, a procedure for homogenization of the laminar two-period composite considered is suggested in [4, 5]. The temperature T of the inhomogeneous body is represented in the form

$$T(r, z, t) = \theta(r, z, t) + h(z)\,\varepsilon(r, z, t)\,, \tag{1}$$

where h is the well-known l-periodic function of the type

$$h(z) = \begin{cases} z - l_1/2\,, & 0 \le z \le l_1\,, \\ -\eta z/(1-\eta) - l_1/2 + l_1/(1-\eta)\,, & l_1 \le z \le l \end{cases} \tag{2}$$

($l = l_1 + l_2$, $\eta = l_1/l$), which satisfies the condition

$$\int\limits_{z-l/2}^{z+l/2} h(z')\,dz' = 0\,, \quad |h(z)| < l\,, \quad 0 \le z < \infty\,. \tag{3}$$

In representation (1) $\theta(\cdot)$ is an unknown function (the macrotemperature), while $\varepsilon(\cdot)$ can be determined by using microlocal thermal parameters associated with the periodicity of the composite-material structure [4].

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Since for the microperiodic composite $|h(z)| \leq l$, then for $0 < z < \infty$, on the basis of relations (2) and (3) at small l, we neglect the second term in the right-hand side of Eq. (1) [4], while in order to find the macrotemperature θ one is to solve the boundary-value problem of heat conduction

$$\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho}\right)\theta + K_0 \frac{\partial^2 \theta}{\partial Z^2} = \frac{\partial \theta}{\partial \widetilde{\mathrm{Fo}}}, \quad \rho, Z > 0, \quad \widetilde{\mathrm{Fo}} < \infty, \tag{4}$$

$$\theta = 0 \quad \text{at} \quad \rho, Z \geq 0, \quad \widetilde{\mathrm{Fo}} = 0, \tag{5}$$

$$\frac{\partial \theta}{\partial Z} = -\Lambda G_i(\rho) \quad (i = 1, 2), \quad \rho > 0, \quad Z = 0, \quad \widetilde{\mathrm{Fo}} > 0, \tag{6}$$

$$\theta \to 0 \quad \text{at} \quad \rho^2 + Z^2 \to \infty, \quad \widetilde{\mathrm{Fo}} > 0, \tag{7}$$

where

$$\Lambda = q_0 a / K, \quad \widetilde{\mathrm{Fo}} = \widetilde{k} t / a^2, \tag{8}$$

$$K_0 = \widetilde{K}/K, \quad K = \widetilde{K} - [K]/\widehat{K}, \tag{9}$$

$$\begin{pmatrix} \widetilde{K} \\ [K] \\ \widehat{K} \end{pmatrix} = K_1 \begin{pmatrix} \widetilde{\alpha}(K^*) \\ [\alpha(K^*)] \\ \widehat{\alpha}(K^*) \end{pmatrix}, \quad \widetilde{k} = k_1 \widetilde{\alpha}(k^*), \tag{10}$$

$$K^* = K_1/K_2, \quad k^* = k_1/k_2. \tag{11}$$

The influence functions in relations (10) have the form

$$\begin{gathered} \alpha(x) = \widetilde{\alpha}(x) - [\alpha(x)]^2/\widehat{\alpha}(x), \quad [\alpha(x)] = \eta(1 - x^{-1}), \\ \widehat{\alpha}(x) = \eta + \eta^2 (1-\eta)^{-1} x^{-1}, \quad \widetilde{\alpha}(x) = \eta + (1-\eta)/x. \end{gathered} \tag{12}$$

We will consider heat fluxes with a uniform

$$G_1(\rho) = H(1 - \rho), \quad \rho \geq 0 \tag{13}$$

or normal (Gaussian)

$$G_2(\rho) = \exp(-\rho^2), \quad \rho \geq 0 \tag{14}$$

spatial distribution of the heat-flux intensity.

2. Determination of the Temperature Field. Having applied to differential equation (4) the Hankel integral transformation with respect to the radial variable ρ, we obtain

$$\frac{\partial^2 \bar{\theta}}{\partial Z^2} - \xi^2 K_0 \bar{\theta} = 0 \,, \tag{15}$$

where

$$\bar{\theta}\,(\xi, Z, \widetilde{\mathrm{Fo}}) = \int_0^\infty \rho \theta\,(\rho, Z, \widetilde{\mathrm{Fo}})\, J_0\,(\xi\rho)\, d\rho \,.$$

The transformed boundary conditions (5)-(7) take the form

$$\bar{\theta}\,(\xi, Z, 0) = 0 \,, \quad Z \geq 0 \,, \tag{16}$$

$$\frac{\partial \bar{\theta}}{\partial Z} = -\Lambda \varphi_i\,(\xi) \quad (i = 1, 2) \,, \quad Z = 0 \,, \tag{17}$$

$$\bar{\theta}\,(\xi, \infty, \widetilde{\mathrm{Fo}}) = 0 \,, \quad \widetilde{\mathrm{Fo}} > 0 \,. \tag{18}$$

Here

$$\varphi_i\,(\xi) = \int_0^\infty \rho\, G_i\,(\rho)\, J_0\,(\xi\rho)\, d\rho \quad (i = 1, 2) \,. \tag{19}$$

For the functions $G_i(\rho)$ of the form given in (13) and (14) the integrals of (19) are calculated analytically [6]. We obtain

$$\varphi_1\,(\xi) = J_1\,(\xi)/\xi \,, \quad \varphi_2\,(\xi) = 0.5 \exp\,(-\,\xi^2/4) \,. \tag{20}$$

In turn, we apply the Laplace integral transformation with respect to the variable t to Eq. (15) and conditions (16)-(18):

$$\bar{\bar{\theta}}\,(\xi, Z, s) = \int_0^\infty \exp\,(-\,st)\, \bar{\theta}\,(\xi, Z, t)\, dt \,.$$

As a result, we come to the second-order ordinary differential equation

$$\frac{d^2 \bar{\bar{\theta}}}{dZ^2} - \beta^2 \bar{\bar{\theta}} = 0 \,, \quad \beta^2 = \xi^2 + \frac{a^2 s}{\tilde{k}} \,, \tag{21}$$

whose solution must satisfy the boundary conditions

$$\frac{d \bar{\bar{\theta}}}{dZ} = -\frac{\Lambda}{s} \varphi_i\,(\xi) \quad \text{at} \quad Z = 0 \quad (i = 1, 2) \,, \tag{22}$$

$$\bar{\bar{\theta}}\,(\xi, \infty, s) = 0 \,. \tag{23}$$

The solution of Eq. (21) subject to conditions (22) and (23) has the form

$$\bar{\bar{\theta}}\,(\xi, Z, s) = \frac{\Lambda}{s\beta} \varphi_i\,(\xi) \exp\,(-\,\beta Z) \quad (i = 1, 2) \,. \tag{24}$$

Since the inverse Laplace transform of the function $(1/s\beta)\exp(-\beta z)$ is equal to [7]

$$\Phi(\xi, Z_0, \widetilde{\mathrm{Fo}}) = \frac{1}{2\xi K_0}\left[\exp(-Z_0\xi)\,\mathrm{erfc}\left(\frac{Z_0}{2\sqrt{(\widetilde{\mathrm{Fo}})}} - \xi\sqrt{(\widetilde{\mathrm{Fo}})}\right) - \right.$$

$$\left. - \exp(-Z_0\xi)\,\mathrm{erfc}\left(\frac{Z_0}{2\sqrt{(\widetilde{\mathrm{Fo}})}} - \xi\sqrt{(\widetilde{\mathrm{Fo}})}\right)\right], \quad Z_0 = \sqrt{K_0}\,Z, \tag{25}$$

then, after applying the inverse Laplace and Hankel transformations successively to relation (24), for the macrotemperature θ we obtain

$$\theta(\rho, Z, \widetilde{\mathrm{Fo}}) = \Lambda\int_0^\infty \xi\varphi_i(\xi)\,\Phi(\xi, Z_0, \widetilde{\mathrm{Fo}})\,J_0(\xi\rho)\,d\xi \quad (i = 1, 2), \quad \rho, Z, \widetilde{\mathrm{Fo}} < \infty. \tag{26}$$

Let us take note of certain important particular cases of the solution (26). If the materials of the conjugate layers are identical, then from Eqs. (9)-(12) we obtain that $K_0 = 1$, $K^* = 1$, $k^* = 1$, $\eta = 0.5$, $\widetilde{\alpha}(\cdot) = \widehat{\alpha}(\cdot) = 1$, $[\alpha(\cdot)] = 0$, and relation (26) coincides with the well-known solution for a homogeneous half-space [1, 8].

In a steady state, when $t \to \infty$, the function Φ of (25) is equal to

$$\Phi(\xi, Z_0, \infty) = \exp(-Z_0\xi)/\xi K_0,$$

and from Eq. (26) we find the temperature distribution in the steady state:

$$\theta(\rho, Z) = \frac{\Lambda}{K_0}\int_0^\infty \varphi_i(\xi)\exp(-Z_0\xi)\,J_0(\xi\rho)\,d\xi \quad (i = 1, 2). \tag{27}$$

The maximum temperature $\theta_{\max}(\rho, Z)$ is attained at the center of the heating spot. When $\rho = 0$ and $Z = 0$, from relation (27) it follows that

$$\theta_{\max} = \frac{\Lambda}{K_0}\int_0^\infty \varphi_i(\xi)\,J_0(\xi\rho)\,d\xi \quad (i = 1, 2), \tag{28}$$

or, with account for the form of the functions $\varphi_i(\xi)$ of (20), that

$$\theta_{\max} = \frac{\Lambda}{K_0} \quad (i = 1, \text{ uniform heating}), \tag{29}$$

$$\theta_{\max} = \sqrt{\left(\frac{\pi}{2}\right)}\,\frac{\Lambda}{K_0} \quad (i = 2, \text{ normal distribution}). \tag{30}$$

For identical materials of the layers at $K_0 = 1$, expressions (29) and (30) coincide with those for a homogeneous half-space [3, 9].

3. Numerical Analysis. The dimensionless spatial variables ρ, Z, the Fourier number $\mathrm{Fo} = k_1 t/a^2$, and the parameters η, K^*, and k^* are the input parameters of the problem. Calculations are carried out for the dimensionless temperature $T^* = T/\Lambda_1$, where $\Lambda_1 = \Lambda\alpha(K^*)$; the results are shown in Figs. 1-5. The solid curves refer to the case of a constant intensity of the heat flux in the form of Eq. (13), and the dashed curves refer to a normal distribution in the form of Eq. (14). Each of the figures presented here consists of parts *a* and *b*. Parts *a* in Figs. 1-4 show the change in T^* with a certain parameter for several values of K^* at $k^* = 1$, and parts *b*, for different values of k^* at $K^* = 1$.

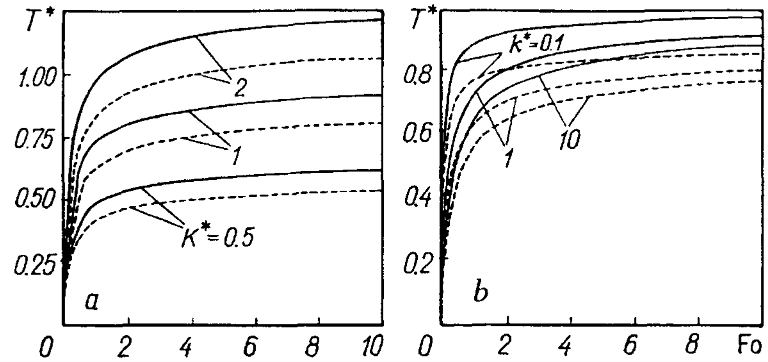

Fig. 1. Change in the dimensionless temperature T^* at the center of the heating circle with the magnitude of the Fourier number Fo at $\eta = 0.5$.

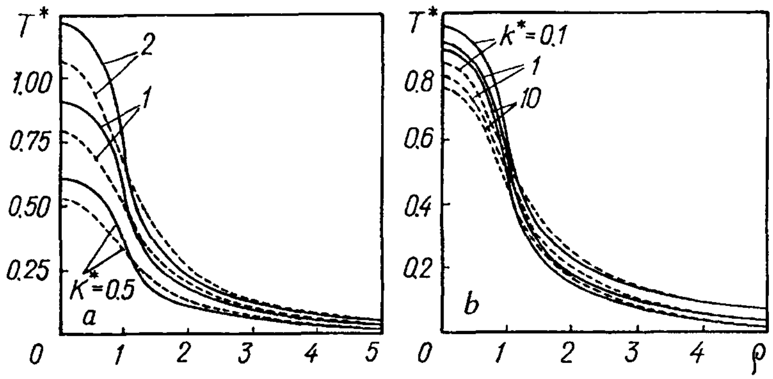

Fig. 2. Change in the dimensionless temperature T^* with the radial coordinate ρ at $Z = 0$; Fo = 10; η = 0.5.

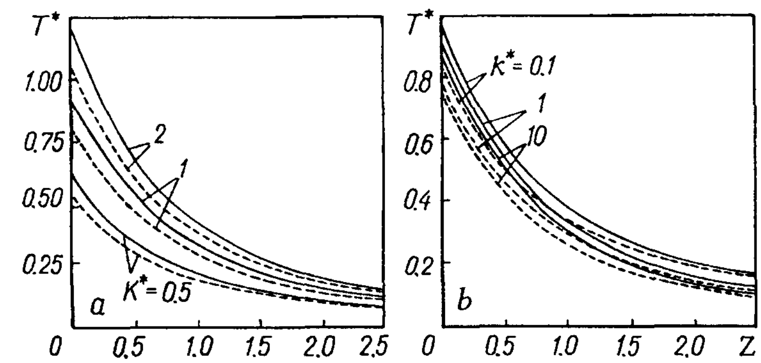

Fig. 3. Change in the dimensionless temperature T^* with the axial coordinate Z at $\rho = 0$; Fo = 10; $\eta = 0.5$.

The duration of transient temperature processes at the center of the heating region ($\rho = 0$, $Z = 0$) is illustrated by the numerical data in Fig. 1. It is seen that the value Fo = 10 corresponds almost completely to the time when the temperature attains a steady state. The temperature at the center of the heating spot for a constant heat flux is always higher than in the case of a normal distribution of it.

The distribution of the temperature T^* along the radius on the boundary surface and along the axis $\rho = 0$ is given in Figs. 2 and 3, respectively. When the heat flux is constant, the temperature field has a substantial gradient near the boundary of the heating circle ($\rho = 1$). The values $\rho = 5$ and $Z = 2.5$ determine the limiting dimensions of the region with a high temperature level.

The influence of the relative thickness of each of the components of the composite (base) layer is shown in Fig. 4. At a prescribed relative thermal diffusivity an increase in the thickness of the component with the greater (smaller) thermal conductivity causes a decrease (increase) in the surface temperature (Fig. 4a). If K^* is fixed,

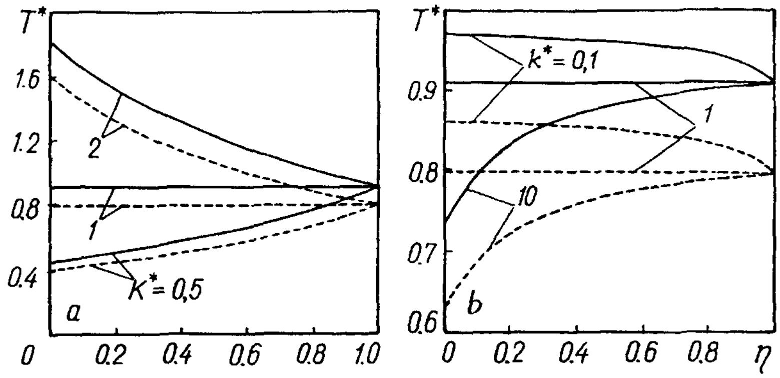

Fig. 4. Dependence of the dimensionless temperature T^* at the center of the heating circle on the relative thickness of the base layer η at Fo = 10.

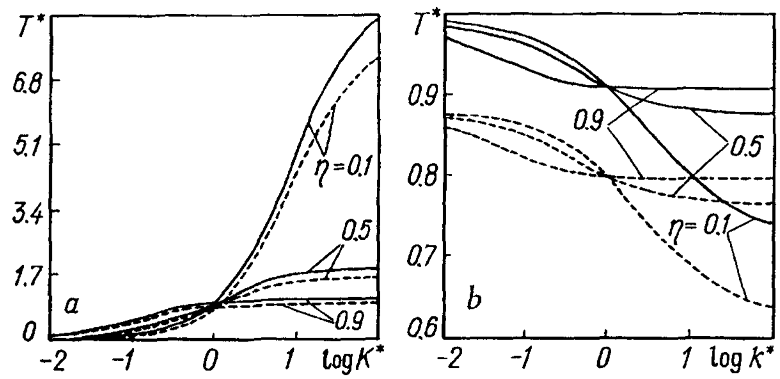

Fig. 5. Dependence of the dimensionless temperature T^* at the center of the heating circle on the relative thermal conductivity K^* (a) and the relative thermal diffusivity k^* at Fo = 10 (b).

then an increase in the parameter η causes an increase in the temperature of the composite surface for $k^* > 1$ and a decrease for $k^* < 1$.

An increase in the relative thermal conductivity K^* at a fixed k^* leads to a rise in the temperature (Fig. 5a). If K^* is fixed, an increase in the relative thermal diffusivity k^* entails a decrease in it (Fig. 5b).

Conclusions. An expression is obtained for the nonstationary temperature at an arbitrary point of a two-period laminar semiinfinite composite when its surface is heated locally by a concentrated heat flux with a uniform or normal distribution of intensity. It is established that:

a) for a the prescribed radius of the heating spot the temperature of the composite surface is always higher for a constant distribution of the heat flux;

b) the temperature fields have considerable gradients the radial and axial directions;

c) an increase in the relative thermal conductivity (K^*) or thermal diffusivity (k^*) exerts a different influence on the temperature of the composite medium, namely, the latter increases with K^* and, conversely, decreases with an increase in k^*.

NOTATION

K_i, k_i ($i = 1, 2$), coefficients of thermal conductivity and thermal diffusivity of the conjugate layers of the composite, respectively; l_i, thickness of the layers; l, thickness of the base layer; θ, macrotemperature; ε, microlocal parameter; T, temperature; r, z, spatial coordinates; a, radius of the heating spot; $\rho = r/a$; $Z = z/a$; t, time; q_0, heat-flux intensity at the center of the heating region; $\mathrm{erfc}(\cdot) = 1 - \mathrm{erf}(\cdot)$; $\mathrm{erf}(\cdot)$, probability integral; $H(\cdot)$, Heaviside unit function; $J_0(\cdot)$, $J_1(\cdot)$, first-order Bessel functions.

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